GEORGIA INSTITUTE OF TECHNOLOGY

OFFICE OF CONTRACT ADMINISTRATION

PROJECT ADMINISTRATION DATA SHEET

[X] ORIGINAL

[ ] REVISION NO. ____

Project No. E-21-E39

DATE: 5/14/81

Project Director: Dr. K. R. Davey

School/Lab E/E

Sponsor: Naval Coastal Systems Center, Panama City, FL 32407

Type Agreement: Contract No. N00612-79-8004, Delivery Order No. HR-39

Award Period: From 4/23/81 To 10/27/81 (Performance) — (Reports)

Sponsor Amount: $10,946

Contracted through: GTRI/~~GIT~~

Cost Sharing: N/A

Title: Three Dimensional Field Analysis of Sources Imbedded in Conducting Media

ADMINISTRATIVE DATA

OCA CONTACT William F. Brown

1) Sponsor Technical Contact: Mr. Jerome Barnes, Naval Coastal Systems Physics Division, Panama City, FL 32407

2) Sponsor Admin./Contractual Contact: Office of Naval Research - Resident Representative 206 O'Keefe Building, Georgia Institute of Technology, Atlanta, GA 30332

Reports: See Deliverable Schedule

Security Classification: ____

Defense Priority Rating: DO-C9

RESTRICTIONS

See Attached Government Contract Supplemental Information Sheet for Additional Requiremen

Travel: Foreign travel must have prior approval - Contact OCA in each case. Domestic travel requires sponsor approval where total will exceed greater of $500 or 125% of approved proposal budget category.

Equipment: Title vests with Government; except that items costing less than $1K vest wit GIT upon acquisition if prior approval to purchase is obtained from Contracting Officer.

COMMENTS:

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SR.281 ?

GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

SPONSORED PROJECT TERMINATION

Date: 9/1/81

Project Title: Three Dimensional Field Analysis of Sources Imbedded in Conducting Media

Project No: E-21-E39

Project Director: K. R. Davey

Sponsor: Naval Coastal Systems Center

Effective Termination Date: 10/27/81

Clearance of Accounting Charges: 10/27/81

Grant/Contract Closeout Actions Remaining:

- x Final Invoice ~~and Closing Documents~~
- _ Final Fiscal Report
- x Final Report of Inventions
- x Govt. Property Inventory & Related Certificate
- x Classified Material Certificate
- _ Other ______

Assigned to: Electrical Engineering (School/~~Laboratory~~)

COPIES TO:

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Other: ______

E21-E39

GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL ENGINEERING
ATLANTA, GEORGIA 30332

TELEPHONE: (404) 894- 2902

June 12, 1981

Naval Coastal Systems Center
Code 340C
Panama City, Florida 32407

SUBJECT: Bimonthly Status Report
Project Director, Dr. D. T. Paris
Contract No. N00612-79-D-8004
"NCSC Omnibus R&D Program"
period covered - April 1, 1981 - May 31, 1981

Ladies and Gentlemen:

The subject report is forwarded in conformance with the contract specifications.

Should you have any questions or comments regarding this report, please feel free to contact me at (404) 894-2902.

Sincerely yours,

Demetrius T. Paris
Professor and Director

Distribution

Addressee, 2 copies
cc: Mr. Tom Bryant, ONRRR
Mr. Otis Rodgers

Input from North Carolina State was not received in time to include in tbis bimonthly report.

NAVAL COASTAL SYSTEMS CENTER
OMNIBUS R&D PROGRAM
CONTRACT NO. N00612-79-C-8004

Bimonthly Status Report

Order Number: HR- 39 Title: Three Dimensional Field Analysis of Sources Imbedded in Conducting Media

Task Leader: Kent Davey

Institution: Georgia Tech

A. SUMMARY STATEMENT OF WORK COMPLETED DURING THE PAST TWO MONTHS

The complete two dimensional problem of a conducting line source in a circular conducting region has been formulated in both an exact boundary value approach and the more versatile integral technique. Both have been computerized and are being compared analytically. Convergence of the two solutions occurs for small distances away from the source. Dominant terms have been cited as giving rise to this convergence; discrepancies arise near interfacial discontinuities when the dominance of the correct first order terms disappear. The need for inclusion of trigonometric terms accounting for direction cosines of interfacial normals has been realized. The integral formulation of singularities has been compared to an approach by Stratton and agreement is established.

B. WORK SCHEDULE STATUS

The two dimensional analysis has been more difficult than we reckoned; this has hindered progress into the three dimensional mode.

C. BRIEF STATEMENT OF PLANNED WORK FOR THE NEXT TWO MONTHS

The main goal is to perfect the 2-D analysis as shown by a convergence of the solution on the exact boundary value approach. A three dimensional problem involving 2 components of the vector potential has been identified and will resume upon completion of the 2-D problem.

D. PROBLEM AREAS

The only underlying problem is the evaluation of singularities; we are still not sure if they are the root of the difficulty.

E. FUNDS EXPENDED

To Date: $3,447

This Two Month Period: $3,447

Funds Remaining: $7,499

Percent of Funds Expended: 31%

Percent of Task Completed: 31%

E21-E39

GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL ENGINEERING
ATLANTA, GEORGIA 30332

ELEPHONE: (404) 894- 2902

August 7, 1981

Naval Coastal Systems Center
Code 340C
Panama City, Florida 32407

SUBJECT: Bimonthly Status Report
Project Director, Dr. D. T. Paris
Contract No. N00612-79-D-8004
"NCSC Omnibus R&D Program"
period covered - June 1, 1981 - July 31, 1981

Ladies and Gentlemen:

The subject report is forwarded in conformance with the contract specifications.

Should you have any questions or comments regarding this report, please feel free to contact me at (404) 894-2902.

Sincerely yours

Demetrius T. Paris
Professor and Director

Distribution:

Addressee, 2 copies
cc: Mr. Tom Bryant, ONRRR
Mr. Otis Rodgers

NAVAL COASTAL SYSTEMS CENTER
OMNIBUS R&D PROGRAM
CONTRACT NO. N00612-79-C-8004

Bimonthly Status Report

Order Number: HR- 39 **Title:** Three Dimensional Field Analysis of Sources Imbedded in Conducting Media

Task Leader: Kent Davey

Institution: Georgia Institute of Technology

A. SUMMARY STATEMENT OF WORK COMPLETED DURING THE PAST TWO MONTHS

The application of the integral technique has been applied successfully to the problem of a line source in a conducting cable as well as a vertical current dipole in a conducting half plane. Agreement in both computer simulations with analytical solutions is realized. The latter problem of the current dipole is a significant step since it is the first time a 3-Dimensional solution has been attempted. Convergence of the solution with an open-ended grid is encouraging. A technical paper on this aspect of the work is completed except for the data and curve plots. The problem of a horizontal dipole in a conducting half space has been formulated using an electric potential $\bar{H} = \bar{T} \cdot \nabla\Psi$.

B. WORK SCHEDULE STATUS

A quasi-3D problem has been solved. The generalized 3D analysis is yet to be done.

C. BRIEF STATEMENT OF PLANNED WORK FOR THE NEXT TWO MONTHS

Work schedule:

1. Complete IEEE paper
2. Analyze generalized 3D attack to ascertain boundary condition problems
3. Formulate equivalent magnetic equivalent circuit to handle "internal" conducting regions with intricate boundaries.

D. PROBLEM AREAS

Matching boundary conditions for the generalized 3-D problem is the greatest single problem. This difficulty has motivated the equivalent magnetic circuit approach.

E. FUNDS EXPENDED

To Date: $5,082

This Two Month Period: $3,446

Funds Remaining: $5,864

Percent of Funds Expended. 46%

Percent of Task Completed: 44%

E-21-E39

DETERMINATION OF TWO AND THREE DIMENSIONAL FIELDS OF SOURCES IMBEDDED IN CONDUCTING MEDIA VIA THE FREDHOLM INTEGRAL TECHNIQUE

ABSTRACT

A numerical integral technique is formulated for determining the magnetic field created by current sources in conducting media. Considered are two examples each involving a current dipole imbedded in a finite region of conducting space. The second example offers a contribution in the area of grid determination and solution convergence; the interface is infinite in extent and must be subdivided as appropriate in any three dimensional problem.

INTRODUCTION

The integral approach has recently received considerable attention in solving magnetic field problems [1, 2, 3]. The attractive feature of the technique is the manner in which equivalent induced sources are lumped onto the boundaries of a conducting region. Upon determination of these equivalent surface sources, the equivalent bulk field calculation is straightforward, requiring little computer time. As opposed to the finite element method which is especially suited to bounded problems having intricate boundaries, the integral technique has as its forte' the world of unbounded discontinuous conducting regions.

BASIC DEVELOPMENT

By way of generalization, we seek a development of the field expression in a non-quasistatic regime in which both source and induced conduction currents exist. Assuming the time variation $e^{j\omega t}$ for all fields we have

$$\nabla \times \bar{E} = -j\omega\mu\bar{H} \tag{1}$$

$$\nabla \times \bar{H} = (\sigma + j\omega\varepsilon)\bar{E} + \bar{J}_s \tag{2}$$

with the non-standard adoption of the vector potential A, where

$$\nabla \times \bar{A} = \bar{H} \tag{3}$$

it follows that

$$\nabla \times \bar{E} = -j\omega\mu(\nabla \times \bar{A}) \tag{4}$$

and

$$\bar{E} = -j\omega\mu A - \nabla\phi \quad . \tag{5}$$

Substitution of (5) into (2), and adopting the guage $\nabla \cdot \bar{A} = -(\sigma + j\omega\varepsilon)\phi$, the standard Helmholtz equation follows

$$\nabla^2\bar{A} - k^2\bar{A} = -\bar{J}_s \tag{6}$$

where

$$k^2 = j\omega\mu\sigma - \omega^2\mu\varepsilon$$

In the examples to follow, the source current is a dipole located at the origin, $I\delta(x,y)$. The appropriate equation to be solved then is

$$\nabla^2\bar{A} - k^2\bar{A} = -I\delta(x,y) \tag{7}$$

Consider the Green's function displaced from the origin at position (ξ,η), $G(x - \xi,\ y - \eta)$ which satisfies

$$\nabla^2 G - k^2 G = -\ \delta(x - \xi,\ y - \eta) \tag{8}$$

~~In~~ A cylindrical, two dimensional problem can be written in terms of the modified Bessel function as

$$G = \frac{1}{2\pi} K_o(kr) \qquad , \tag{9}$$

whereas in a three dimensional problem

$$G = \frac{e^{-kr}}{4\pi r} \tag{10}$$

Multiplying (7) by G, (8) by A, subtracting and integrating over a homogeneous region of volume V gives

$$\iiint(A\nabla^2 G - G\nabla^2 A)dv - k^2\iiint(AG - GA)dv$$

$$= -\iiint A\delta(x - \xi,\ y - \eta)dv + \iiint GI\delta(x,y)dv \tag{11}$$

The first term can be reduced to a surface integral using Green's identity; this yields the result that the bulk ~~A's~~ vector potential depends on the surface values and their normal derivatives, i.e.,

$$A(\xi,\eta) = IG(\xi,\eta) - \iint\left(A\frac{\partial G}{\partial n} - G\frac{\partial A}{\partial n}\right)ds \tag{12}$$

Thus, given the A's and their normal derivatives, the vector potential at any interior location is known. Numerically this is best done by subdividing the surface into elements each having area ΔS and constant values A, $\frac{\partial A}{\partial n}$. (This last assumption is not necessary; more accurate results follow by assuming linear or parabolic fits [4], reducing the number of elements). With S total surface elements,

$$A(\xi,\eta) = IG(\xi,\eta) - \sum_{j=1}^{S}\left(A_j\frac{\partial G}{\partial n} - G\frac{\partial A_j}{\partial n}\right)\Delta S \tag{13}$$

The calculation of the A_j's and $\frac{\partial A_j}{\partial n}$'s are obtained by writing 13 ~~(15)~~ for ξ,η on the surface for all the regions of interest and then matching the interfacial fields. This is best demonstrated by example.

<u>CASE I</u> - The Two Dimensional Field Generated by an Infinite Line Current in a Conducting Coaxial Cable.

In this first example we seek the field generated by the line current imbedded in the conducting coaxial cable of Figure 1. The vector potential $\bar{A}$ is z directed and the Green's function is

$$G(x-\xi,\ y-\eta) = \frac{1}{2\pi}\ K_o\left(k\sqrt{(x-\xi)^2 + (y-\eta)^2}\right)$$

$$= \frac{1}{2\pi}\ K_o(kr) \tag{14}$$

The normal derivative of G is also needed in (13); this is evaluated as

$$\frac{\partial G}{\partial n} = \nabla G \cdot \hat{n} = \frac{\partial G}{\partial r}\,\hat{a}_r \cdot \hat{n} = -\frac{k}{2\pi} K_1(kr) \cos \Psi \tag{15}$$

For region 1

$$A_1(\xi,\eta) = \frac{I}{2\pi} K_o(kr_o) + \oint \frac{k}{2\pi} A\, K_1(kr) \cos \Psi \, d\ell + \frac{1}{2\pi} \frac{\partial A}{\partial n} K_o(kr)\, d\ell \tag{16}$$

where $\quad r_o = \sqrt{\xi^2 + \eta^2}$

$$r = \sqrt{(x-\xi)^2 + (y-\eta)^2}$$

The line integrals are a result of the two dimensionality of the problem. For field points $A(\xi,\eta)$ in region 2, the corresponding integral becomes

$$A_o(\xi,\eta) = -\oint \frac{k_o}{2\pi} A\, K_1(k_o r) \cos \Psi \cdot d\ell - \frac{1}{2\pi} \oint \frac{\partial A}{\partial n} K_o(k_o r)\, d\ell \tag{17}$$

where $k_o = \sqrt{j\omega\mu\sigma_o - \omega^2 \mu \varepsilon_o}$

The sign change is consistent with the normal for region 0 being reversed.

The next step is to let the point (ξ,η) approach the boundary (ξ', η') for both regions. In doing so, it is helpful to break the surface integrals into segments not containing the field point (ξ', η') and a segment that does (designated $B-\Delta B$ and ΔB respectively. Equation (16) becomes

$$A_1(\xi', \eta') = \frac{I}{2\pi} K_o(kr_o) + \frac{k}{2\pi} \int_{B-\Delta B} A\, K_1(kr) \cos\Psi \, d\ell$$

$$+ \frac{k}{2\pi} \int_{\Delta B} AK_1(kr) \cos\Psi \, d\ell + \frac{1}{2\pi} \int_{B-\Delta B} \frac{\partial A}{\partial n} K_o(kr) d\ell$$

$$+ \frac{1}{2\pi} \int_{\Delta B} \frac{\partial A}{\partial n} K_o(kr) d\ell \tag{18}$$

A limiting integration process is sufficient to show that the fifth term of (18) is zero and the third is $\frac{1}{2}A(\xi', \eta')$ [5,6]. It follows that the ith element vector potential is

$$A_{i,1}(\xi', \eta') = \frac{I}{\pi} K_o(kr_o) + \frac{k}{\pi} \sum_{\substack{j=1 \\ j\neq i}}^{S} A_i K_1(kr) \cos\Psi \, \Delta\ell$$

$$+ \frac{1}{\pi} \sum_{\substack{j=1 \\ j\neq i}}^{S} \frac{\partial A_1}{\partial n} K_o(kr) \Delta\ell \tag{19}$$

or in region 2,

$$A_{i,2}(\xi', \eta') = - \frac{k_o}{\pi} \sum_{\substack{j=1 \\ j\neq i}}^{S} A_i \, K_1(k_o r) \cos\Psi \, \Delta\ell$$

$$- \frac{1}{\pi} \sum_{\substack{j=1 \\ j\neq i}}^{S} \frac{\partial A_i}{\partial n} K_o(k_o r) \, \Delta\ell \tag{20}$$

Equations (19) and (20) constitute 2S equations and 4S unknowns. The additional 2S equations are supplied via the boundary conditions

2-S 4-S 4S

$$\hat{n} \cdot ||\bar{B}|| = 0 \quad \rightarrow \quad \mu_o A_o = \mu_1 A_1 \Big|_{\rho=a} \tag{21}$$

$$\hat{n} \times ||\bar{H}|| = 0 \quad \rightarrow \quad \frac{\partial A_o}{\partial \rho} = \frac{\partial A_1}{\partial \rho} \Big|_{\rho=a} \tag{22}$$

The equations (19-22) are most easily solved in matrix form on the computer.

The exact boundary value approach yields the solution

$$A_1 = \frac{I}{2\pi} K_o(kr) + B_1 I_o(kr) \tag{23}$$

$$A_o = B_o K_o(k_o r) \tag{24}$$

where

$$B_1 = \frac{I}{2\pi} \frac{\{K_o(k_o a)k K_1(ka)\mu_o - K_o(ka)k_o K_1(k_o a)\mu_1\}}{\mu_o K_1(k_o a)kI_1(ka) + k_o K_1(k_o a)I_o(ka)\mu_1}$$

$$B_o = \mu_1 \frac{I}{2\pi} \frac{\{K_o(ka)kI_1(ka) + k K_1(ka)I_o(ka)\}}{\mu_o K_1(k_o a)kI_1(ka) + k_o K_1(k_o a)I_o(ka)\mu_1}$$

INSERT 1 HERE!

Insert I

Figure 2 compares the exact boundary value solution with the solution generated by the boundary integral equation (BIE) technique. The parameters used in the analysis are frequency $f = 1Hz$, coaxial radius $a = 10$ meters, line current $I = 1amp$, and conductivity $\sigma = 1000$ mhos/meter. The boundary for the BIE technique is divided into N segments. A study of the curves in Figure 2 indicates good agreement between the two techniques except near the boundary, where the solutions begin to diverge. The divergence is caused by the arguments of the modified Bessel functions found in the kernel of (17) getting smaller: that is, the field point approaches a source point which is located on the boundary. Cases were run for N = 360, 720 and 2880. As N is increased, the accuracy likewise is increased as noted in Figure 2a. ~~For the~~ N = 2880 ~~run~~, the difference between the two techniques is less than 1% for values of R up to 9.95 meters.

When

CASE II - Vertical Current Dipole Imbedded in an Infinite Half Conducting Space

The three dimensional Green's function relevant to this three dimensional problem shown in Figure 3 is

$$G = \frac{e^{-kr}}{4\pi r} \tag{25}$$

where $r = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z-\phi)^2}$

the directional derivative can be written

$$\frac{\partial G}{\partial n} = \nabla G \cdot \hat{n} = - \frac{e^{-kr}}{4\pi r} (k + \frac{1}{r}) \cos \psi \tag{26}$$

The equivalent representation of A from (12) is

$$A_1(\xi, \eta, \phi) = \frac{I\, e^{-kr_o}}{4\pi r_o} + \iint A_1 \frac{e^{-k_1 r}}{4\pi r} (k_1 + \frac{1}{r}) \cos \psi \, dS + \iint \frac{\partial A}{\partial n} (\frac{e^{-k_1 r}}{4\pi r}) \, dS \tag{27}$$

and in region 0

$$A_o(\xi, \eta, \phi) = \iint \left\{ A_o \frac{e^{-k_o r}}{4\pi r} (k_o + \frac{1}{r}) (-\cos\psi) \, dS + (- \frac{\partial A_o}{\partial n}) \frac{e^{-k_o r}}{4\pi r} \right\} dS \tag{28}$$

Note ~~here~~ unlike case I, here the source dipole I has the units of amp-meters.

Now we let the field point (ξ, η, ϕ) approach the surface (ξ', η', ϕ'). As was done in case I, the integrals may be evaluated over segments, ΔS, containing the field point and over those that do not, $S - \Delta S$, to get the "self" terms. Only the $\frac{1}{r^2}$ term survives the ΔS segment integration giving the value $\frac{1}{2} A(\xi', \eta', \phi')$. With this result and the observation that $\psi = 90°$ on the surface, it follows that for a total of S surface elements with area ΔS, the expressions for A on the surface are

$$A_{o,i}(\xi', \eta', \phi') = - \sum_{\substack{j=1 \\ j\neq i}}^{S} \frac{\partial A_{oj}}{\partial n} \frac{e^{-k_o r'}}{2\pi r'} \Delta S_j \tag{29}$$

$$A_{1,i}(\xi', \eta', \phi') = \frac{I\, e^{-kr_o'}}{2\pi r_o'} + \sum_{\substack{j=1 \\ j\neq i}}^{S} \frac{\partial A_{ij}}{\partial n} \left(\frac{e^{-kr'}}{2\pi r'}\right) \Delta S_j \tag{30}$$

where $r_o' = \sqrt{\xi'^2 + \eta'^2 + (h)^2}$; $r' = \sqrt{(x-\xi')^2 + (y-\eta')^2 + (z-\phi')^2}$

Equations (29) and (30) yield 25 equations and 45 unknowns. With the boundary requirements tangential H continuous and normal B continuous come the additional 25 conditions

$$A_1 = A_o \tag{31}$$

$$\frac{\partial A_1}{\partial n} = \frac{\partial A_o}{\partial n} \tag{32}$$

at the interface. The equation to solve are written as follows

$$\begin{bmatrix} L_A \\ L_B \end{bmatrix} [A] + \begin{bmatrix} K_A \\ K_B \end{bmatrix} \left[\frac{\partial A}{\partial n}\right] = \begin{bmatrix} B_A \\ B_B \end{bmatrix}$$

where $L_B = L_A = \begin{cases} 1 & i=j \\ 0 & i\neq j \end{cases}$

$$K_A = \begin{cases} 0 & i=j \\ \dfrac{-k_o^2}{k^2} \dfrac{e^{-k_o r'}}{2\pi r'} \Delta S_j & i\neq j \end{cases}$$

$$K_B = \begin{cases} 0 & i=j \\ \dfrac{e^{-kr'}}{2\pi r'} \Delta S_j & i \neq j \end{cases}$$

$$B_A = 0$$

$$B_B = I\frac{e^{-k_1 r_o'}}{2\pi r_o'}$$

Optimum solution is achieved by recognizing the symmetry of the problem, i.e., the constancy of A and $\frac{\partial A}{\partial n}$ on concentric rings centered at the origin. A and its normal derivative are sought on these rings, th~~i~~us reducing the unknowns in such a problem significantly (Figure 3b).

INSERT 2 HERE!

Insert 2

Both the number of rings N and the number of points per ring M are variable. An approach for determining both of these factors for a convergent solution is shown in Figure 4. The curves in this figure were generated to determine a convergent field solution at the field point X = 10. m, Y = 0, Z = 5 m. First, values for both N and M are arbitrarily chosen and held constant. Then Rlimit is varied. Rlimit is the outside radius of the last ring considered in the analysis (see Figure 3b). Field contributions from rings beyond this limit are considered negligible. Rlimit is varied until a null occurs in the field data as shown in Figure 4. The value of Rlimit at the null represents the maximum value needed for convergence. For values of Rlimit smaller than this value, truncation error occurs since non-negligible field contributions are truncated. For values of Rlimit greater than the null value, discretization error occurs since the boundary discretization averages the fields over too large of a surface area. Once Rlimit is determined at the null, the next step is to increase both N and M until the fields converge to a constant value. This may also require slight adjustments in Rlimit. Once a constant value of the field is determined, then the problem is solved. the constant field should accurately represent the desired solution at the field point of interest. Figure 4 shows such a solution for the vertical dipole of Figure 3a. The result converges to approximately the same value as that predicted by an image solution technique.

CONCLUSION

The integral technique for solving field problems is quite useful indeed. OK Its ~~effica~~cy both in a two and three dimensional realm has been demonstrated. Of course, the next step in demonstrating the generality of the approach is to apply it to a problem where more than one component of the vector potential is involved. Such an analysis in general however involves mixed boundary conditions.

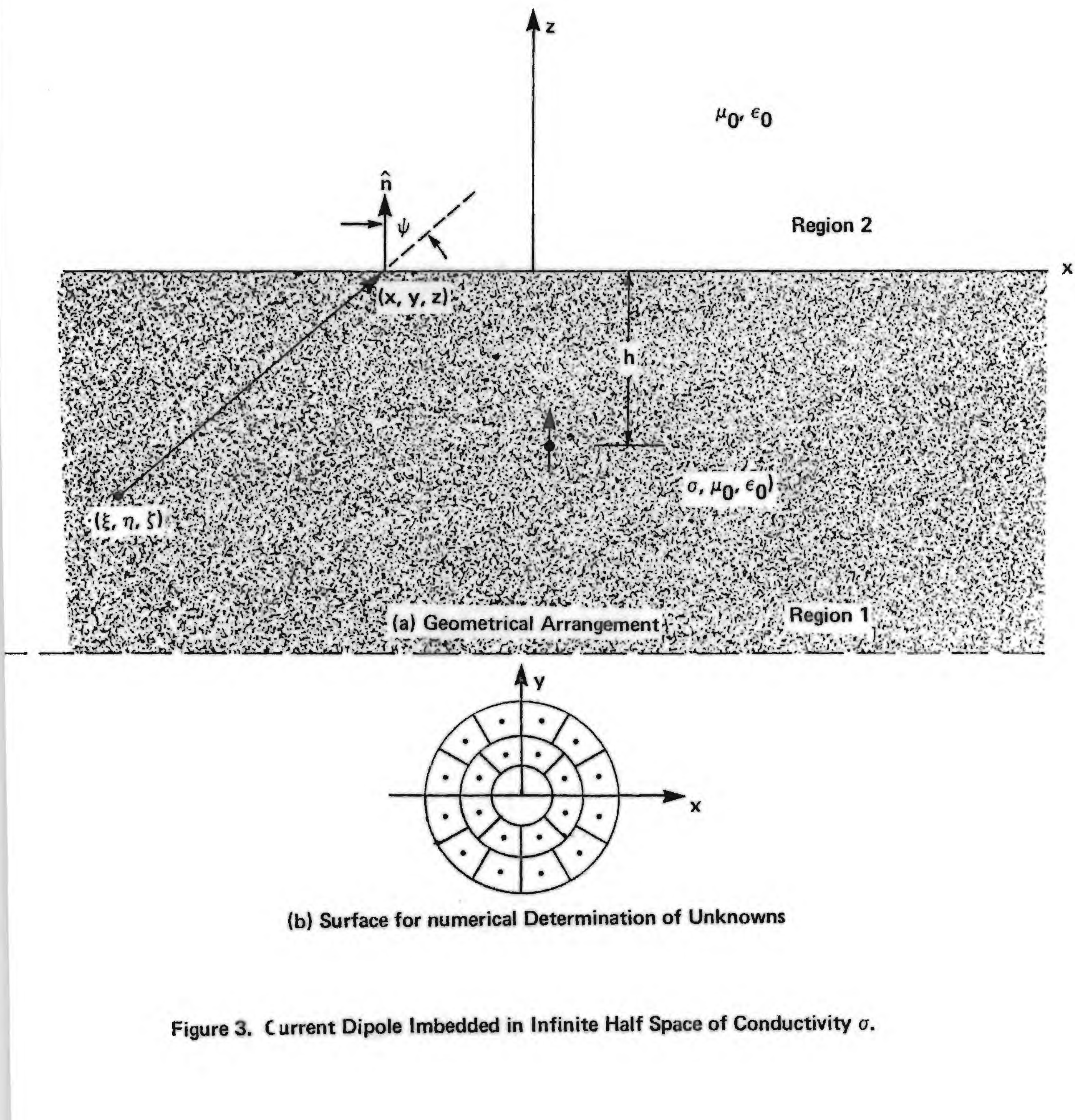


Figure 3. Current Dipole Imbedded in Infinite Half Space of Conductivity σ.

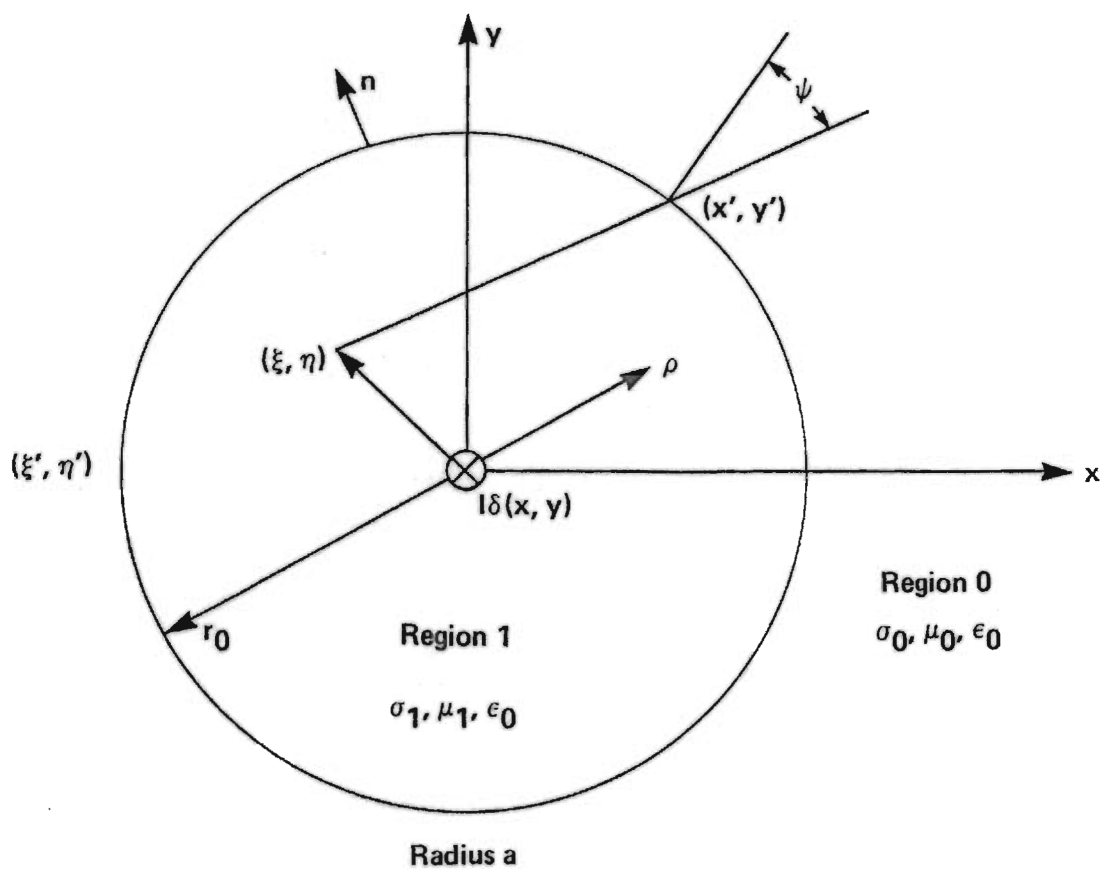


Figure 1. Infinite Line Current in a Conducting Coaxial Region.

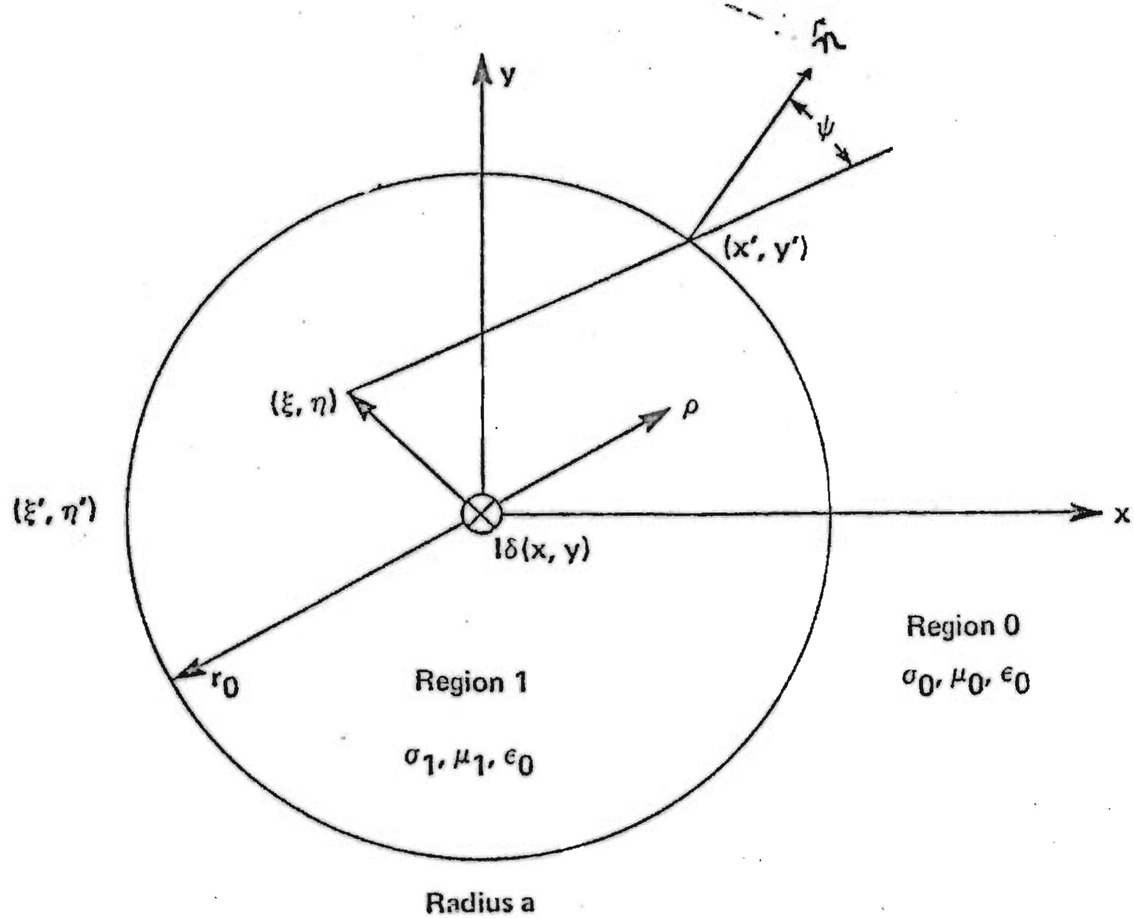
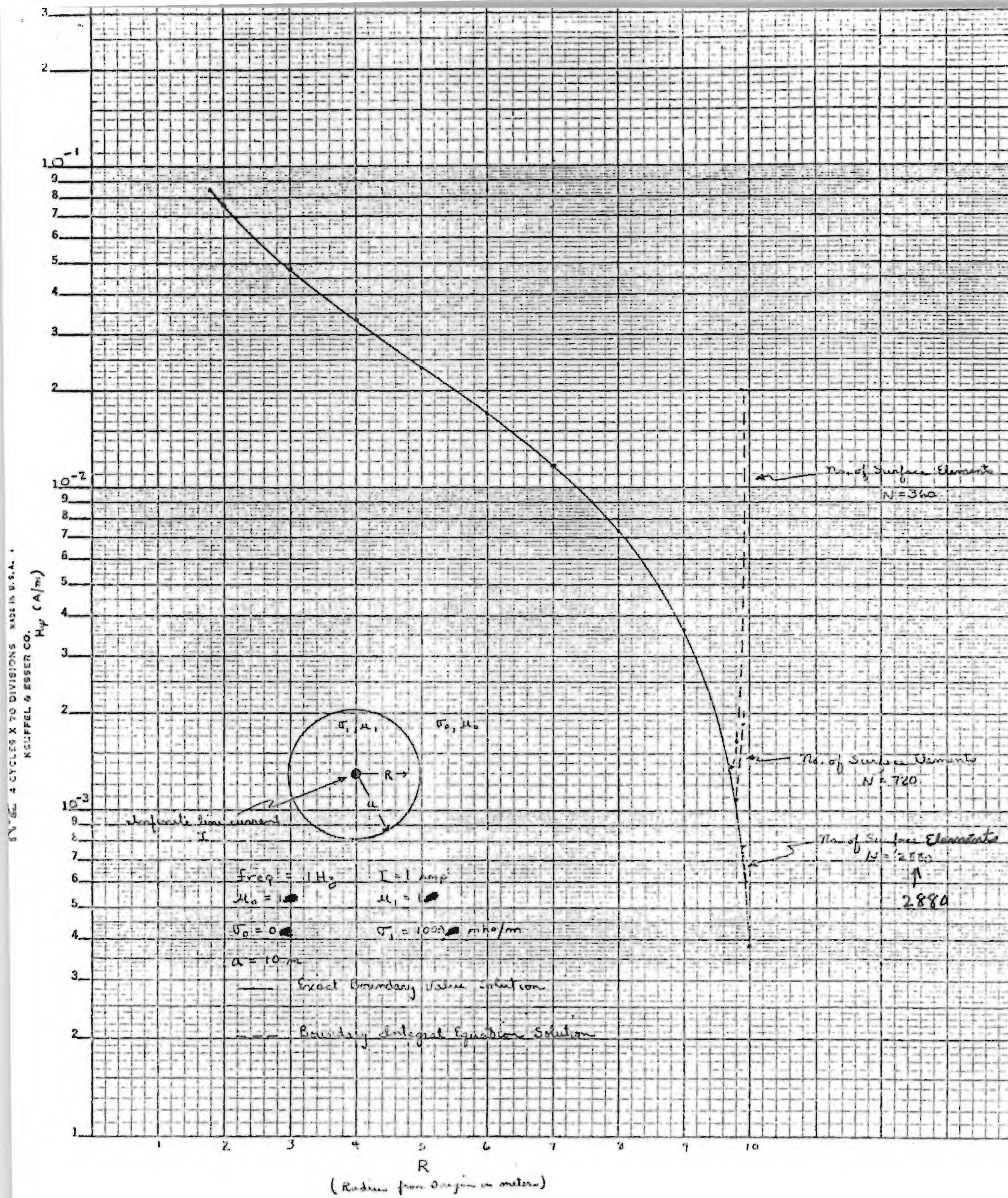
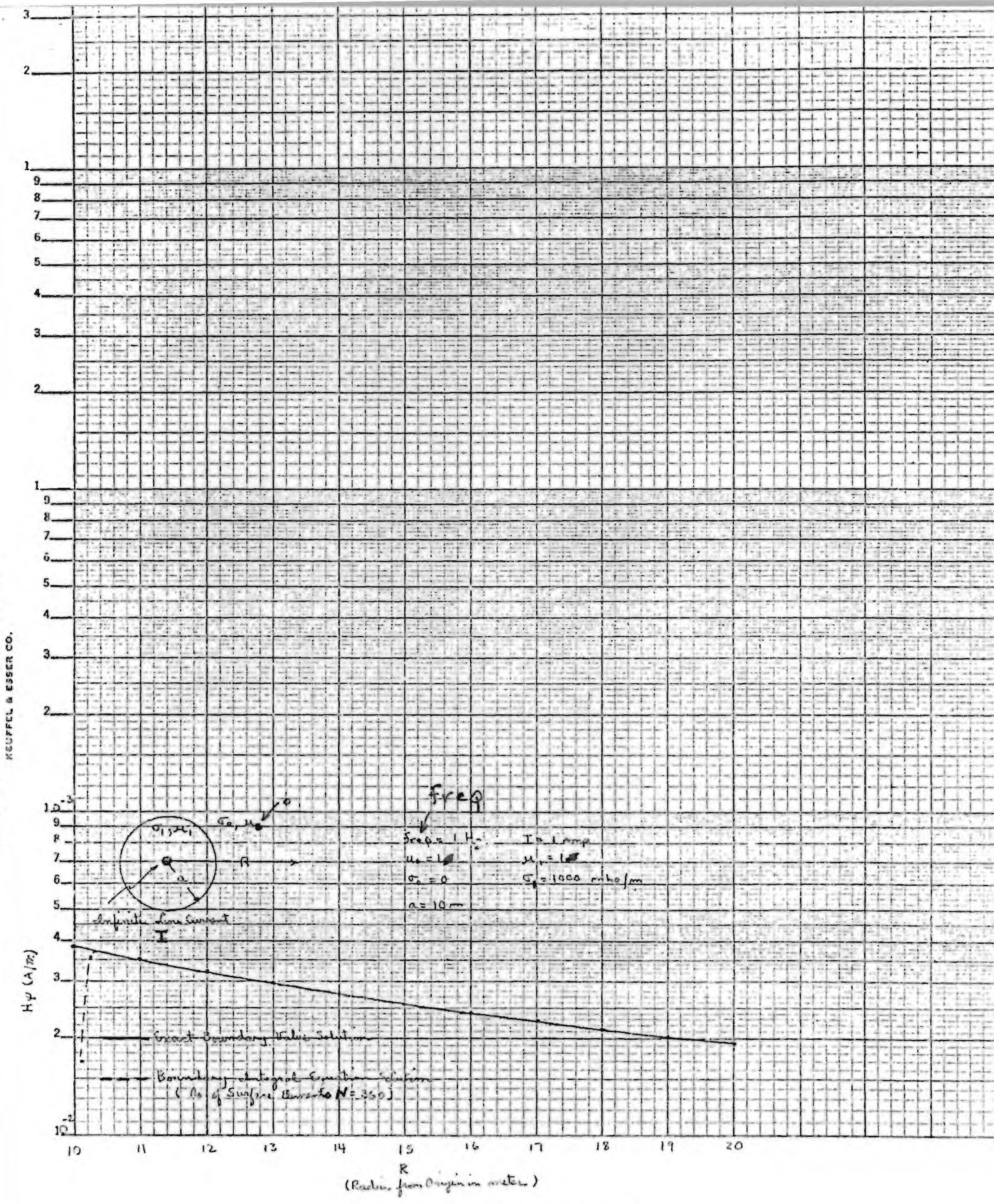


Figure 1. Infinite Line Current in a Conducting Coaxial Region.

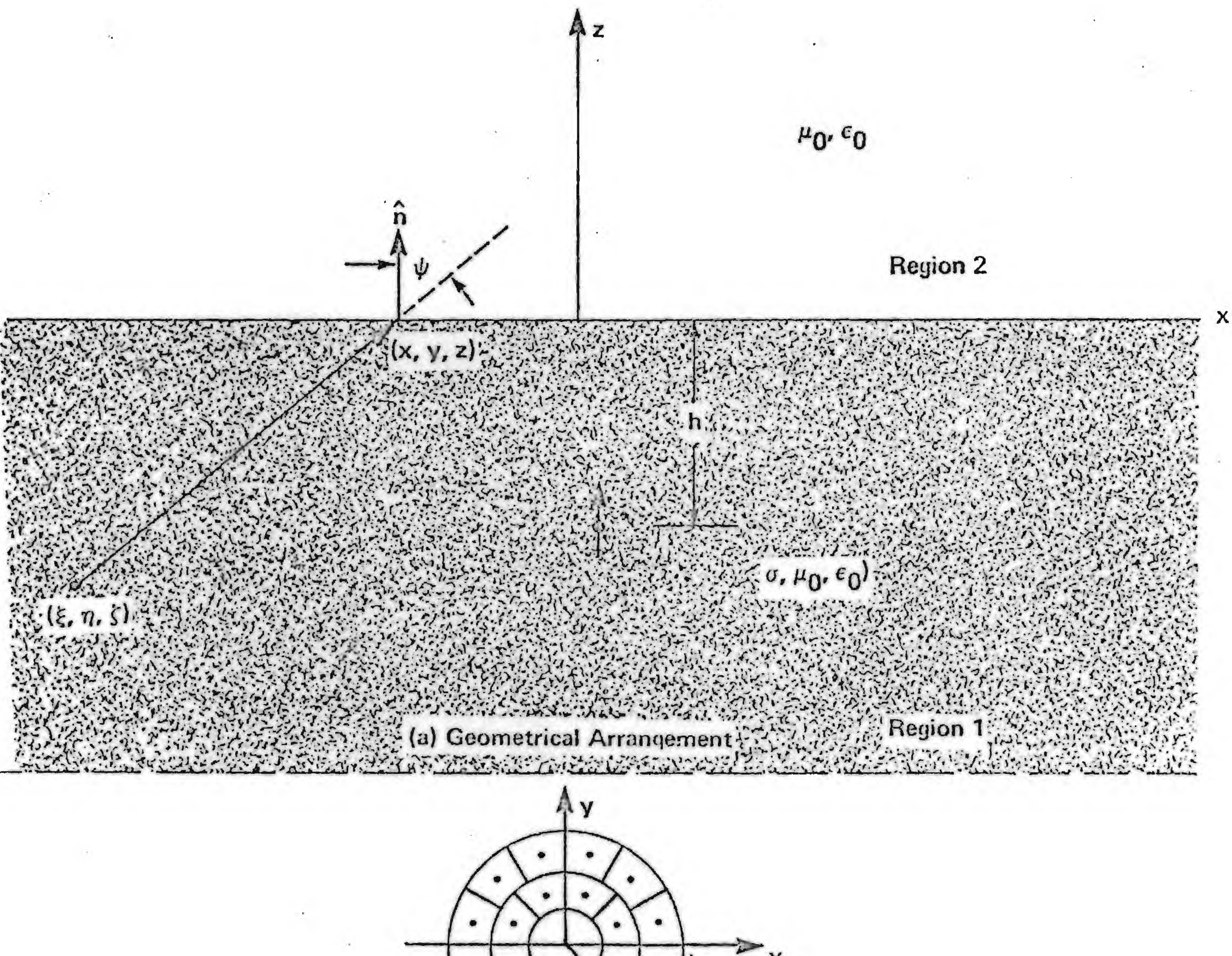


(a) R from origin to boundary.

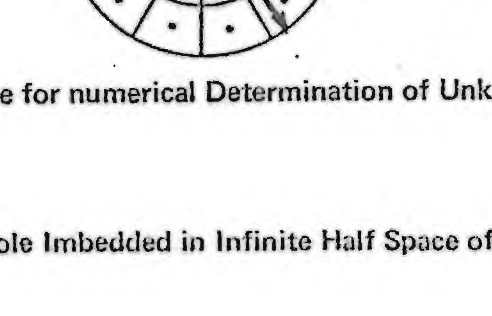


(b) R from boundary [redacted] outward.

Figure 2. Exact Boundary Value Technique versus Boundary Integral Equation Technique for Infinite Line Current located in a Coaxial Conductor.



(a) Geometrical Arrangement



(b) Surface for numerical Determination of Unknowns

Figure 3. Current Dipole Imbedded in Infinite Half Space of Conductivity σ.

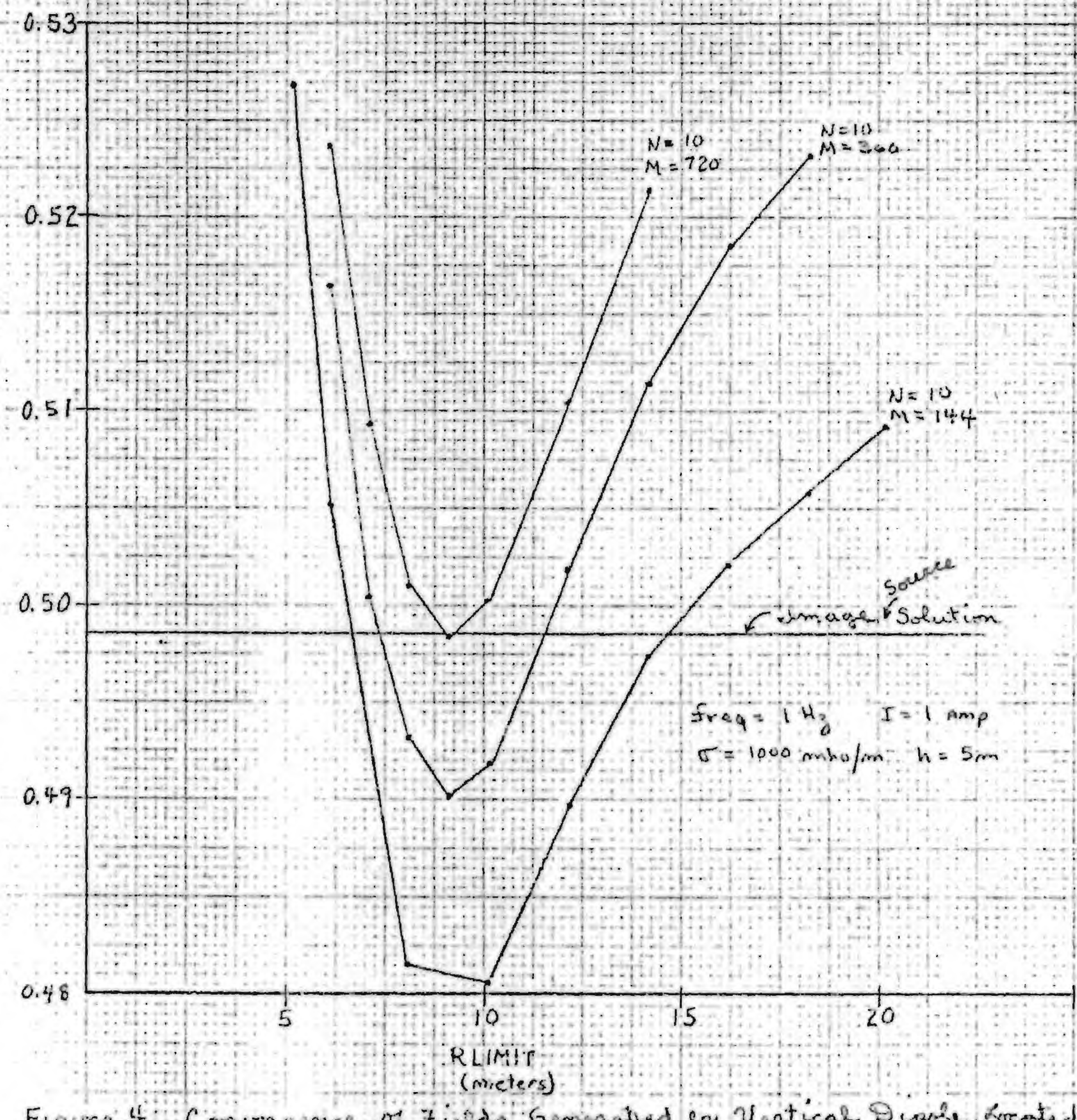


Figure 4. Convergence of Fields Generated by Vertical Dipole Located in a Semi-infinite Conducting Half-Space With the Field Point Located at X = 10 m, y = 0, z = 5 m.